700

VOLUME 81 SEPARATE No. 700

PROCEEDINGS

AMERICAN SOCIETY
OF
CIVIL ENGINEERS

MAY, 1955

PERMEABILITY, PORE PRESSURE AND UPLIFT IN GRAVITY DAMS

by Roy W. Carlson, A.M. ASCE

POWER DIVISION

{Discussion open until September 1, 1955}

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Printed in the United States of America

Headquarters of the Society
33 W. 39th St.
New York 18, N. Y.

PRICE $0.50 PER COPY

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This paper was published at 1745 S. State Street, Ann Arbor, Mich., by the American Society of Civil Engineers. Editorial and General Offices are at 33 West Thirty-ninth Street, New York 18, N. Y.

PERMEABILITY, PORE PRESSURE AND UPLIFT IN GRAVITY DAMS

Roy W. Carlson,[1] A.M. ASCE

SYNOPSIS

Results are presented of studies on the intensity of pore pressure in the bodies of concrete dams, and of the resulting uplift force. The low permeability of concrete, even with as little as two bags of cement per cubic yard, indicates that equilibrium pore pressure should never develop in a service dam. The effect of air entrainment is to reduce the penetration of pore pressure into a dam to a fraction of the penetration without air. An approximate solution for the percentage area subject to uplift pressure is offered, in which the area is expressed in terms of known voids and of the ratio of the elastic modulus of the solid constituents of the concrete to that of the composite concrete. The effective area is deduced to be nearly 100 per cent of the gross area in the lean concretes investigated. However, in view of the low permeability, high intensity of pore pressure need never develop in the interior of a gravity dam, if reasonable precautions are taken. Current design practice may, therefore, be overly conservative. Also, expansive strains due to saturation and pore pressure are reported. These strains tend to increase the compressive stress near the upstream face and further improve the safety of a dam.

INTRODUCTION

One of the most doubtful steps in the design of a gravity dam is the allowance to be made for uplift pressure. The actual uplifting force due to water in the pores always has been an uncertain factor. Not only has there been doubt about the intensity of pressure in the pores of the concrete, but also there has been doubt about the effective area over which that pore pressure acts. The present discussion does not concern the foundation, which is set aside as a separate problem.

The intensity of pore pressure in a dam depends mainly upon the permeability of the concrete and the length of time the reservoir remains filled. It depends also upon such factors as drainage and location of joints or cracks. Even without drainage or joints, it may take centuries of full reservoir for equilibrium to be established such that there is a linear distribution of pore pressure from heel to toe. And yet, most designs are based upon some sort of equilibrium condition prevailing. Thus, the intensity of pore pressure to be expected in a particular dam seems to be worthy of study.

Differences of opinion have been aired for at least 20 years as to the effective area over which pore pressure acts, and many conflicting statements appear in print. At first thought, it would appear that simple tests or

1. Cons. Engr., Berkeley, Calif.

measurements could be made to determine the uplift force due to water pressure in the pores of concrete. But the problem is not so simple, because as long as the concrete is not jointed and pore pressure varies linearly from heel to toe, the stress distribution is the same when uplift pressure is acting as when there is no pore pressure. For example, suppose a dam is loaded by filling the reservoir so quickly that there is no pore pressure, and suppose that the dimensions and weight are such that the downward pressures are 100 psi. at the heel and 400 psi. at the toe. Now suppose that pore pressure is added of an intensity varying linearly from 200 psi. at the heel to zero at the toe. The net downward force is still 100 psi. at the heel and 400 psi. at the toe, in order to maintain a force balance. But after the pore pressure has been added, the solid framework of the concrete near the heel is subjected to a tension of about 100 psi. of gross area (assuming nearly 100 per cent of effective area) while the compression force in the pores is 200 psi. Should the concrete have been unable to carry the 100 psi. tension, a crack would have formed, and there would surely have been 100 per cent of area subject to pore pressure in the region of the crack and then the stress distribution would have been altered. But stress measurements alone would have given no clue as to the effective area in any part of the dam before failure or in the uncracked portion after failure. The safety of the dam depends upon the effective area even though the stresses may be unaffected. Only through measurements of strains or tests depending upon ultimate strength does there seem to be any hope of determining the effective area on which pore pressure acts.

Current Design Assumptions

The various agencies which are building dams have disagreed rather widely as to the uplift assumptions to be used in designing a gravity dam. The Bureau of Reclamation has sometimes specified that the effective area should be two thirds and the pore pressure should vary linearly from full reservoir pressure at the upstream face to tailwater pressure at the downstream face. The various Districts of the Corps of Engineers have differed among themselves in the past, but quite a number have used the assumptions of 100 per cent area and two thirds (more or less) of full reservoir pressure at the upstream face, diminishing linearly to tailwater pressure at the downstream face. Occasionally, designers take advantage of drainage and assume reduced pore pressure at the location of the drains, the pressure then varying linearly to reservoir pressure at the upstream face and linearly also, but at a lower slope, from the drains to the downstream face. It is doubtful if any current assumptions as to intensity of pore pressure are very near the conditions which exist in the dams. There is less doubt about the area over which pore pressure acts in concrete, but, nevertheless, disagreement exists here also. Therefore, a better understanding of permeability, pore pressure, and resulting uplift pressure might serve to revise design assumptions to a sounder, more consistent, and more economical basis.

Test Program

Commencing late in 1948, conferences were held in the Office of the Chief of Engineers to consider what tests could be made to clarify the question of uplift in gravity dams. At the outset, two phases of the problem were recognized. The intensity of pore pressure as controlled by permeability,

dimensions, duration of full reservoir, etc. was considered to be the more important phase of the problem. The uplift force caused by the pore pressure, and the attendant strains in the concrete, were grouped together as the second and more difficult phase of the problem.

The permeability of mass concrete was believed to be especially important in view of the trend toward lower cement content and the use of air entrainment. All indications were that uncracked, rich concretes are so impermeable that pore pressure cannot develop to anywhere near the equilibrium condition in the body of a dam unless the reservoir is full for centuries. But even lean concretes may be impermeable enough to permit lower pore pressure to be assumed in design. Very little is known about the permeability of lean concretes such as are now being used in gravity dams, especially at greater ages and with air entrainment. The effect of continued saturation also is highly speculative and yet it may be a large factor in reducing the percolation of water into a gravity dam. Consequently, a program of permeability tests was developed to solve the first phase of the problem.

The permeability tests were planned to investigate a variety of concretes with especial emphasis on the effects of (a) cement contents as low as 2 bags per cubic yard, (b) air entrainment, (c) ages up to several years, and (d) continued saturation. Many refinements were introduced to insure that the results would truly reveal the permeability of corresponding concrete in a gravity dam. Specimens were cast in metal containers and turned on their sides before setting in order that the permeability be measured in a direction corresponding to the horizontal direction in a gravity dam. Temperatures were controlled to simulate those in an actual dam. Water was controlled as to air content to prevent abnormal release of air within a specimen, where the pressure drops sharply. The tests were made in the Engineering Materials Laboratory of the University of California under the direction of Professor Raymond E. Davis. A full account of the permeability tests up to the age of three months may be found in a report to the Chief of Engineers, Washington D. C., by Davis entitled "Permeability and Triaxial Tests of Lean Mass Concretes," dated April 1950. Tests at later ages are being made in the Clinton Laboratory of the Corps of Engineers.

In addition to the permeability tests, another program of tests was developed to determine the effective area over which pore pressure acts, and also the strains which are caused by pore pressure. Although most engineers seemed to be convinced that the effective area was certain to be either nearly 100 per cent or exactly 100 per cent, there appeared to be need for experimental verification in the working stress region. Previous tests had indicated roughly 100 per cent of area in specimens at time of failure, but no tests had yet been made to determine the effective area at working stresses. True, some compression tests had been reported which appeared to show that the area was about 100 per cent in specimens subjected to moderate hydrostatic pressure. But current investigations proved that the interpretation of these tests was faulty, since the hydrostatic compressibility was not dependent on pore area but dependent only on the elastic properties of the solid constituents of the concrete. Furthermore, it was highly desirable to know what volume changes were to be expected from saturation and pore pressure in order that any accurate predictions could be made of stress at the heel of a gravity dam.

Many types of test for determining effective pore area were suggested before one finally was adopted. The early preference was for tests of models of gravity dams, but no ready solutions were found for the many difficulties

of obtaining results which would truly be indicative of uplift force. Therefore, the specimen finally adopted was a long cylinder which would be subjected to hydrostatic pressure around the sides, and to mechanical pressure at the ends. Thus, there would be full pore pressure in the central region of the cylinder, diminishing to zero near the ends. Simultaneously an axial load could be applied at the ends of the cylinder, corresponding to expected heel stress in the dam. Strains were measured at the center of the cylinder to indicate change in length whenever hydrostatic or mechanical loads were applied or removed. In order to interpret the data, strain meters were embedded in companion specimens of the permeability series and these specimens would be saturated and subjected to full hydrostatic pressure. A full description of the tests, with results up to the age of three months, is contained in the Raymond E. Davis report of April 1950, mentioned above.

Analytical studies were made concurrently with the laboratory tests. Invaluable help was given by members of the following organizations: Office of the Chief of Engineers, Bureau of Reclamation, Portland Cement Association, and University of California (see acknowledgments hereinafter).

Results of Permeability Tests and Their Implications

Many interesting revelations come from the permeability tests, but only a few need be mentioned here. In the particular mass mixes used, the permeability of 2-bag concrete averaged less than three times that of 3-bag concrete at the age of three months. Thus the average permeability of the 2-bag concretes was only 580×10^{-12} cubic feet per second per square foot for a unit pressure gradient. The same specimens, left saturated, and retested at an age of one year showed a reduction in permeability to a fraction of the three-month value.

Specimens tested at ages of three months, eighteen months and twenty four months showed progressive decrease in permeabilities, the average for 2-bag concretes being only 135×10^{-12} cu.ft. per second at the age of two years. An interesting revelation was that the three-month specimens when retested at one year were as impermeable as companion specimens initially tested at two years. Thus, the saturation and testing at three months apparently hastened the process of hydration and lowering of permeability.

In the very lean mixes, the sand fineness had a large effect upon the permeability; the finer the sand, the lower the permeability. For the 2-bag mix, sand with a fineness modulus of 2.25 and a water cement ratio of 0.86 resulted in a two-year permeability average of 81 (units as above), while a fineness modulus of 2.50 and a water cement ratio of 0.80 gave a permeability average of 190. The finer sand more than offset the effect of the higher water content in this case.

The effect of air entrainment upon permeability was favorable. For mixes having the same cement content, aggregate grading, and consistency, the permeability was 439 for an air content of 6.2 per cent, and 662 for the corresponding mix with no air-entraining agent but with 1.4 per cent of air. Thus the air entrainment reduced the permeability in this particular case by 34 per cent. The mix here was 2.25 bags of cement per cubic yard, the only one where the effect of air entrainment was investigated.

In addition to air entrainment reducing the actual permeability, it delays the penetration of water into a mass of concrete by a much larger additional factor due to the storage capacity for water provided by the voids. In a dam the size of Detroit or Pine Flat, about 20 million gallons of water would be

required to fill the voids produced by 5 per cent of entrained air. Therefore, it may be worthwhile to compute the probable rate of penetration of water into a gravity dam under the pressure of a full reservoir.

As the water front moves farther into the concrete, it may be assumed that the air voids are completely filled by water behind the water front. Since water is relatively incompressible, very little additional water is needed to accommodate the reduction in volume as the pressure increases behind the advancing water front. Therefore, it can be assumed that a linear pressure gradient will prevail from reservoir level at the upstream face to zero at the farthest depth of water penetration, provided that the reservoir level remains constant. Under these assumptions, the following equation expresses in feet the distance D penetrated by water in T days into a mass of concrete where the voids are 5 per cent, the water head is 400 feet, and the permeability is 200 in customary units:

$$D^2 = 0.2767T$$

The permeability of 200 was chosen for this example because test results up to the time this is written indicate it to be about the one year value for concrete containing only 2 bags of cement per cubic yard. Of course, the permeability would be greater at earlier ages, but the reservoir is not likely to be filled before at least a year after the concrete near the foundation is placed, and the permeability may be much lower at the ages indicated for deep penetration of water into the dam. Furthermore, this equation applies only to the case where there is no relief by internal drains or ungrouted joints.

Figure 1 shows the penetration of water into a 400-ft. dam for various durations of full reservoir as computed by the equation above. This figure applies only to the continuous concrete of the dam, and would be modified greatly near the base of a dam whose foundation is permeable. It would be modified considerably, also, if there were no air entrainment. After 200 years of full reservoir, the indicated penetration is less than half the thickness of the dam. Under the conditions of this example (no joints or drains, no reduction in permeability after the age of one year, and no drying from the downstream face) the water should appear at the downstream face in somewhat more than 1000 years. Of course, this applies only to sound concrete and does not consider the foundation to be permeable. But it does definitely indicate that, as far as the concrete itself is concerned, low internal pore pressures are likely to prevail either with or without drainage, and that only slight drainage would be effective against so slow a percolation rate.

The above considerations of permeability have neglected the effects of capillarity. An investigation of the computed and observed rates of penetration of water into the permeability specimens indicated that the rates were approximately as expected without any allowance for capillarity. However, it must be recognized that an abnormally large gradient existed in the specimens and that the effect of capillarity would, therefore, need to be very large to be evident. An investigation of the quantitative effect of capillarity is sorely needed.

Uplifting Force Produced by Pore Pressure

The problem of determining the area subject to pore pressure was hampered by erroneous beliefs of long standing and by the inherent complexity of

a material like concrete. It was I. L. Tyler of the Portland Cement Association who disproved the long-published statement that the hydrostatic compressibility of concrete is related to the effective pore area. And it was Douglas McHenry of the Bureau of Reclamation who uncovered fallacies in several analyses which were developed by individuals and offered in review. But many others helped in the stumbling progress toward a partial clarification of the problem of determining effective pore area.

When the test program was planned, it was believed that the only difference between a specimen subjected to full hydrostatic pressure and one with hydrostatic pressure around the sides only (ends open), would be the uplift forces due to the pore pressure being zero at the ends in the latter case. Thus, it was specified that a simple subtraction of the strains in the two corresponding specimens would give the strain due to uplift. This strain was to be converted into uplift force by determining the axial load required to produce an equal strain. Later, it was proved that the difference was bound to yield 100 per cent of "effective" area if the measurements were accurate. It became apparent that the step of going from a specimen loaded hydrostatically all around, to a specimen loaded hydrostatically on the sides only, amounted to removing the load both from the "effective" and from the "non-effective" area at the ends of the first specimen. Thus, the test program could not yield the effective area of the mass concretes in the simple manner which had been anticipated.

Using the observed strains as a guide, an approximate solution of the effective pore area of concrete was developed. The concrete was idealized to the extent of assuming most of the solids to be accumulated into cubes, regularly arranged, and connected by the remainder of the solids. At first thought, this might seem to be an intolerable departure from actual concrete, but relative dimensions instead of being chosen at random were computed to make all pertinent properties identical with those of the concrete. Thus, the idealized concrete could be computed to have the correct void content, the correct modulus of elasticity of solid constituents (provided they are uniform), the correct modulus of elasticity of the concrete as a whole, the correct Poisson's ratio for the solid constituents, and approximately the correct Poisson's ratio for the concrete as a whole. It can be shown that an idealized concrete made up of a porous solid of uniform elastic properties will have a Poisson's ratio of from about one half to two thirds that of the solid material, purely by virtue of the geometry. Thus, this idealized concrete is an accurate "particular" case of concrete although it evidently is not a general case.

The mechanics of the uplifting force in concrete have been clarified somewhat by the simplified analysis presented in the appendix. For example, it can be proved that the uplift force on any section depends only upon the pore pressure and area at that section, and not upon the area at other sections. Of course, there may be other forces applied in a dam, such as those due to temperature effects, water loads and gravity. However, what we define as uplift is simply a matter of pore pressure and area at that section. This is contrary to a common belief about forces in specimens such as those tested by Davis. It has sometimes been declared that the uplift on a section in the central region where there exists a uniform pore pressure, not only is caused by the diminution in pore pressure near the ends, but is dependent upon the effective pore area near the ends where the pore pressure diminishes. It can be proved that no matter what the pore area may be in the regions where pore pressure drops to zero, as long as that pressure does drop to zero, the uplift on a section at midlength is the same. Also, it no longer is considered

sound to think of uplift as causing an overall pull on the concrete. As an illustration, turn again to the long cylinder specimens subjected to pore pressure only in the midlength region. Assume, for example, that there may be a solid lens across the specimen, this lens having substantially no pore area. Such a condition is approached by taking a section through a maximum area of aggregates and other solids. By treating the half cylinder from such a section to one end (where forces are zero) as a free body, it is seen that there is no force on this section. This proves that there is no overall pull on the specimen, but suggests instead that the uplift is produced locally by the pore pressure acting as a jack within the pores. As a contrast to the assumed solid section, one might find a section with a full 100 per cent of pore area, such as a joint. Here the uplift would act over the whole area, and this would be the worst section, quite independent of pore areas elsewhere.

The actual areas disclosed by the tests and idealized analysis were not far from 100 per cent for the lean concretes used. The sectional porosity, after hydration, was near 15 per cent for all of the concretes containing entrained air. The ordinary modulus of elasticity varied from approximately 4×10^6 psi. for the 2-bag concrete, to $5 \cdot 8 \cdot 10^6$ for the 3-bag concrete. The modulus of elasticity of the solids was found to be approximately 17×10^6 psi. Thus, the ratios of moduli were 4.25 and 2.92, respectively. The corresponding areas can be found from the curves of Figures 2A and 3A in the Appendix. They range from 97 to 98 per cent, the lower area being for the richer mix.

A surprising revelation came from applying the area equation to minerals. The physical properties of most aggregates indicate them to possess nearly 100 per cent of effective area. Very little published information is available to show the relative compressibility of minerals with and without impermeable cover to provide the E_c and E_p values needed for the equation. However, an article by W. A. Zizman in the Proceedings National Academy of Sciences 1933, Volume 19, entitled "Compressibility and Anisotropy of Rocks At and Near the Earth's Surface," lists these data for a dozen rocks. For all rocks, except obsidian, the compressibility due to fluid pressure when covered was much greater than when uncovered. Therefore, it is evident that the equation, developed in the Appendix, will yield a large fractional area, especially since the sectional porosity is small in most rocks.

For example, Quincy granite had a compressibility four times as great with an impermeable cover as it had without the cover. This means that the modulus of elasticity of the solid crystals was about four times that of the composite granite. Assuming the porosity to be 1 per cent by volume, the value of "f" is seen to be about 0.004 from Fig. 3. A glance at Fig. 4 shows that the effective pore area would be more than 99 per cent if the rock structure simulates the idealized porous solid envisioned for concrete.

Apparently, the pores in most minerals exist as poor contacts between crystals because, if the small volume of pores existed as well distributed spherical cavities, for example, no such large differences in elastic moduli could be expected.

Strains Due to Saturation and Pore Pressure

An important contribution of the laboratory investigations, mentioned above and conducted by Raymond E. Davis, is the revelation of actual strains or dimensional changes which accompany saturation and pore pressure. The

results are such as to make the handling of strains quite simple, thanks largely to the low compressibility of the solid constituents of the concrete, and to a compensating expansion of the concrete due to saturation. The measured expansion of the concrete in the long cylinders, from the saturated state without pore pressure to the addition of 200 psi. of pore pressure (free ends), was from 5 to 10 micro-inches per inch less than for an ordinary tensile stress of 200 psi. This is explained by the facts that, while the solid constituents are being compressed by the pore pressure and there is only a small average uplift force, most of the strain occurs in the filler material between the larger particles, and this is under nearly 200 psi. tensile stress. The compensating factor regarding the strains is the effect of saturation. An average of results from strain measurements on permeability specimens, obtained prior to their regular testing, showed an expansion of 7 micro-inches per inch due to saturating from a mass-cured condition. Adding this to the strain due to pore pressure as mentioned above, one finds a total expansive strain which is almost exactly what one would expect if the uplift were a mechanical tension of numerically equal amount. The designer can make good use of this result in computing the modification of stress due to restraint near the upstream face of a dam when the pore pressure penetrates only a moderate distance. Also, the engineer who analyzes measured strains in a structure should be gratified to learn that the strains due to pore pressure and saturation can be treated as mechanical strains. Only if the pore pressure differs greatly from the 200 psi. here considered, or if the concrete were not of the lean mixes tested for this study, would a deviation from the simple treatment be necessary.

CONCLUSIONS

1. The permeability of well-designed mass concretes, even with mixes as lean as 2 bags of cement per cubic yard are so low as to indicate that equilibrium pore pressures are not likely to be achieved in a moderate number of decades, if ever.
2. The effect of entrained air is found not only to reduce the permeability of concrete but also to delay the penetration of water into a dam by a factor of several times, by virtue of the water capacity of the air voids.
3. The effective area over which pore pressure acts in concrete can be estimated from the relative compressibilities of sealed and unsealed specimens subjected to fluid pressure.
4. The maximum effective pore area to be expected in the lean concretes tested in the current program varies from 97 per cent in the 3-bag mix to 98 per cent in the 2-bag mix.
5. The effective pore area in most common aggregates appears to be more than 90 per cent.
6. The expansion due to saturation and pore pressure is a large factor in reducing likelihood of high tensile stress near the upstream face of a gravity dam, as long as the pore pressure is localized near the face.

RECOMMENDATIONS FOR FURTHER WORK

In the course of this investigation, the lack of information on many factors loomed up repeatedly. Following are some of the subjects suggested for

additional testing or investigation:

1. Permeability at Greater Ages. It is highly speculative to compute the penetration of water into a concrete dam for decades when the permeability is known only up to the age of one year. Provision should be made for tests up to at least ten years of age.

2. Permeability of Concrete Continuously Saturated. The reduction in permeability of concrete is likely to be influenced greatly by continued hydration, and the continued hydration, in turn, may be expected to be affected by the availability of water in the pores. A beginning on this subject is now being made at the Clinton Laboratories of the Corps of Engineers.

3. Permeability of Concretes Containing Pozzolans. Since some evidence indicates a many-fold reduction in permeability due to the inclusion of active siliceous materials in the concrete, this subject gives promise of a large reduction in permeability and consequent pore pressure, at low cost.

4. Permeability of Concretes Containing Different Cements and Aggregates. Sweeping conclusions cannot be drawn from permeability tests on a single cement and aggregate. Different types of cements and aggregates should be "tied-in" with the current investigations.

5. Physical Properties of Aggregates. More information is needed on the hydrostatic compressibility of aggregates, both sealed and unsealed, and on the Poisson's ratio of the minerals under the two conditions.

6. Extraneous Deformations in Concrete Subjected to Hydrostatic Pressure. The hydrostatic compressibility of concrete is so small that any simultaneous deformation, such as might be due to gel expansion caused by squeezing water into unfilled pores, could make a serious error in the interpretation. A better understanding of the effect of water pressure on concrete is needed.

7. Compressibility of Aggregates Due to Water Pressure. The reported tests on compressibility of minerals were made on very small specimens with kerosene as the fluid. Corresponding tests are needed on larger specimens subjected to water pressure.

8. Effect of Capillarity on Percolation of Water. In view of the large theoretical effect of capillarity on the percolation of water through concrete, tests should be made to determine the rates of penetration of water into concrete specimens where the pressure gradient is about the same as in a typical dam.

ACKNOWLEDGMENT

This report, although written by R. W. Carlson, comprises the work of the following engineers: R. F. Blanks and Douglas McHenry of the Bureau of Reclamation; T. C. Powers and I. L. Tyler of the Portland Cement Association; B. W. Steele, F. B. Slichter, J. R. Gilbert, D. A. Buzzell, and R. A. Barron of the Office of the Chief of Engineers; and R. E. Davis of the University of California.

APPENDIX

Particular Example Showing Relation Between Strains and Pore Area

It is desired to derive an expression for the effective pore area in terms of some factor which can be measured. Up to the present time, the

deformation of the concrete (or strain) appears to be the only measurable action which is closely related to uplift forces. Therefore, the aim is to derive an equation for effective pore area in terms of measured strains.

Consider first an "idealized" porous solid which has many of the characteristics of concrete, before becoming involved and possibly confused by the general treatment of uplift forces. Suppose the idealized concrete to be like the section shown in Figure 1A, where most of the solid particles have been collected into cubes and the remainder, together with the pores, comprise a filler material between cubes. Let the entire specimen be a large sphere and the unit cell be exactly one cubic inch as depicted in Figure 1A. Merely for convenience, we have chosen the solid cube to be 0.9 inch on a side, leaving 0.05 inch all around for filler material. For further simplicity, the filler material is pictured as fibers or columns between cubes and it has a sectional area of exactly 0.1 square inch per gross square inch. Here we have a case where the total pore space can be calculated to be 24.4 per cent of the volume, the void area on a section through the solid cube is 19 per cent (chosen so as to miss the columns of filler material) and the void area on a section between cubes is 90 per cent.

We can arrive at an expression for "percentage effective uplift area" by three steps as follows:

1. Write an expression for stress and consequent strain due to the application of a hydrostatic load on the spherical specimen of which the 1-inch cube is an interior, unit cell. For this first step, assume the specimen to have its surface sealed against entrance of pressure into the pores, or else assume that the specimen is large enough to prevent development of pore pressure in the interior. If the test actually is made by the application of water pressure to the surface of the sphere, then some relief must be provided for the pressure which will develop in the pores due to volume change, because the equations will assume zero pore pressure for this step. Actually, it is simpler to make the test by applying an axial load to a cylindrical specimen, and computing the corresponding strain for triaxial load (hydrostatic) by applying a double Poisson's ratio correction, one for load in each of the other two directions. The reference condition (zero strain) requires that the specimen be fully saturated but under no stress of any kind. This applies to the succeeding two steps as well.

2. Next write an expression for the strain due to hydrostatic pressure when the pressure has penetrated the pores and is applied to the surface as well.

3. Finally, express the strain after a quick removal of the hydrostatic pressure from the surface of the sphere, with the pore pressure within the sphere remaining as in the second step. The internal pore pressure is assumed to remain unchanged in this step, neglecting the effect of volume change, because it is desired to simulate the static condition in a dam.

Now, it can be shown that the change from step 2 to step 3 amounts to removing the pressure from 100 per cent of the surface area, and since step 1 amounts to applying the pressure to 100 per cent of the area, a subtraction of the strain of step 3 from that of step 2 should equal the strain of step 1, except for being opposite in sign.

Now, let us proceed to write these expressions for a case where the modulus of elasticity and Poisson's ratio of the solid material are known or assumed to be 8 million psi. and 0.25 respectively. For this example, the solid columns in the filler material are assumed to have the same elastic properties. But to make the case a little more general, let us pretend that this was

not known but that a measurement indicated the modulus of elasticity of the gross filler material to be 0.8 million psi., this being merely the result of having one tenth sectional area. For further simplification, let us write all expressions for unit pressure, one psi., and let us be concerned mainly with vertical stresses in our unit cell.

Step 1. The stress due to 1 psi. surface pressure is simply 1 psi. in all directions, with no pore pressure. The contribution to vertical strain by the filler material is equal to the stress (1) times the length (0.05 above and 0.05 below, or 0.1 inch) and divided by E (0.8 million psi.) and the area (1 square inch). The Poisson's ratio effect is considered to be negligible in the filler material as far as this strain is concerned, since pressure is not applied to the sides of the filler elements. The contribution to vertical strain by the solid cube is the stress (1) times the length (0.9 inch) divided by E (8 million psi.) and the area (0.81 square inch). Now the Poisson's ratio effect reduces this strain by 25 per cent for the stress in each lateral direction, so the strain can simply be multiplied by 0.50 to take account of Poisson's ratio. Thus, we have for the total vertical strain in step 1

$$e_1 = -\frac{0.1}{0.8} - \frac{0.5 \times 0.9}{8 \times 0.81} = -.1250 - .0693 = -.1943 \times 10^{-6}$$

Step 2. Hydrostatic pressure in the pores, as well as at the surface, produces a compressive strain which is independent of the pores. It is merely the stress (1) divided by E (8 million psi.), corrected for Poisson's ratio by subtracting 25 per cent for each lateral direction. Thus, the strain in step 2 is

$$e_2 = -\frac{0.5}{8} = -.0625 \times 10^{-6}$$

Step 3. Quick removal of pressure from the surface of the sphere produces somewhat more complex stresses. Let "n" be the fractional area subject to pore pressure in the filler material. Then the stress in the filler material is found by balancing the forces on a plane through the material and parallel to a face of the cube. The product of "n" and the pressure (1 psi.) must equal the tensile force per gross square inch of filler, since the external force is now zero. Thus, the force in the filler is simply "n" psi., reckoned on the full area. Now the strain due to this force is "n" times the length (0.1 inch) divided by E (0.8 psi.). But this must be corrected for the additional expansion of the fibers due to water pressure around them, and this is the pressure (1 psi.) times the length (0.1 inch) times 0.50 (for the double Poisson's ratio) and divided by E (8 million psi. since we are dealing here with solid fibers). In addition, there is a slight tension in the solid cube which can be found to be 0.19 lb. by balancing the forces on a plane through the cube. The contribution to strain by this tension in the cube is 0.19 times the length (0.9 inch) times 0.50 to allow for the double Poisson's ratio correction and divided by E (8 million psi.) and divided also by the area (0.81 square inch). The terms for strain in step 3 are then

$$e_3 = \frac{0.1n}{0.8} + \frac{0.1 \times 0.50}{8} + \frac{0.19 \times 0.5 \times 0.9}{8 \times 0.81} = .1250n + .0062 + .0132$$

Inserting numerical values in the expression stated above, namely

$e_3 - e_2 = e_1$, there is obtained

$$-1250n + .0062 + .0132 + .0625 = .1943$$

and $$n = .1124/.1250 = 0.90$$

Thus, it is seen that the effective pore area in the filler material (worst section) might have been found from an analysis of measured strains to be 90 per cent, as it should be in this known case. This means that the specimen should fail when subjected to internal pore pressure (only) of an intensity about 10 per cent above the tensile strength. However, it may be noted that the stress in the aggregate particles (cubes) is low.

It is interesting to examine the stress conditions on a practical scale before proceeding further. Suppose that the pore pressure were 200 psi. as at the base of a 460-foot dam, instead of 1 psi. Then the tensile stress in the fibers of the filler material of this idealized example must be 200 x 10 or 2000 psi. In any tensile loading of concrete, the stresses must be magnified similarly in those elements which support the stress applied to the gross section in porous regions.

Now the stress in the solid cube (simulating the aggregate) was found to be very low. In this example where the volume of voids was 24 per cent and rather high for concrete, the tensile stress in the aggregate was found to be only 47 psi. (.19 lb. times 200 and divided by 0.81). Of course, this value cannot be applied directly to concrete, because we have deliberately made the volume percentage of voids higher than that which would be deduced by taking a section through the aggregate. But a fact which is illustrated by the example is that uplift force does not produce an overall pull on the concrete nor is the tension anywhere near uniform from point to point. The tension is highest across sections where the void areas are greatest, which is not really surprising. Another fact illustrated by this example and consistent with strain studies on actual concrete is that the tensile strains due to the uplift force of pore pressure are almost as great as though there were actually an overall pull on the concrete of the full amount of the "effective" uplift. This is merely a consequence of the relatively high modulus of elasticity of the solid constituents of the average concrete. The filler material (with its voids) is responsible for most of the deformation.

Development of Equation for Uplift Area

The nature of concrete is such that an exact mathematical expression for uplift cannot be developed from only those physical properties ordinarily known. For example, it is clear that any particle of aggregate which is unbonded over its entire surface should be reckoned as part of the voids because it can take part in no structural action. Although aggregate particles are not likely to be unbonded over their entire areas, occasionally one may find particles to be unbonded over substantial parts of their areas and, in such cases, they would act partly as voids. Also, there are minor Poisson's ratio effects and property differences which are indefinite. Therefore, it appears inadvisable to strive for an exact mathematical expression for uplift area.

The area which we are searching is that of a section which passes through the largest possible number of voids without deviating too far from a plane section. This may be termed the "worst section" and one which yields the

largest area. The requirement that it may not deviate too far from a plane section is simply to avoid introducing shearing strength, for we are interested only in the area of pore pressure which must be supported by tensile strength, or which produces tensile strain. It is fairly evident that an average plane section will have the same void area as the concrete as a whole, or about 15 per cent for ordinary mass concrete with air entrainment. Thus, the worst section is a special one and highly localized. When dealing with a heterogeneous material like concrete, the worst section in one region may be somewhat different than that in another region. It should be realized that strain measured on a 10-inch gage length spans many worst sections, as well as many sections with low void areas. It appears to be expecting too much, therefore, for measured strains to reveal the one worst section. In the idealized analysis which follows, the intention is to reveal the worst plane section that might be expected with an arrangement of particles which maintains all of the pertinent, known properties of the concrete.

It may be possible that a fairly dependable solution of the effective-area problem can be obtained by considering the concrete to be idealized as in the foregoing example. That is to say, assume most of the solid matter to be collected into cubes, with the rest of the solids together with the voids making up a porous filler material between cubes. The exact amount of filler material can be computed to satisfy the conditions, assuming that only that part of the solid matter which can be treated as columns between cubes shall be assigned to the filler. The volume of voids must be known and the amount of solids in the filler is then computed so as to account for the modulus of elasticity of the concrete being lower than that of the solids. Thus, Figure 2 has been prepared to provide a means of determining effective thickness of filler material in terms of known ratio of elastic moduli and of known void content of a particular concrete. In this treatment, it is assumed that all solids have the same modulus of elasticity and the same Poisson's ratio, and this may be nearly true since voids are responsible for the large variations in elastic moduli among rocks. It may be seen from the development of the equation for uplift area that the Poisson's ratio for composite concrete does not enter. However, a calculation will show that a reasonable ratio, considerably lower than that for the solids, in a natural consequence of the separate treatment of solids and paste (or filler).

The following is a short derivation of an equation for the effective uplift area "n." Since this derivation is exactly parallel with the example already given, little further explanation is necessary. It is gratifying that the final result is independent of Poisson's ratio, since one would expect this if all ingredients have the same inherent properties.

Development of More General Equation for Effective Uplift Area

Let f = linear amount of filler per inch (between cubes)
F = " " " solid " "
P = pore pressure, psi.
u = Poisson's ratio of all solid material
E_x = modulus of elasticity of filler material (reckoned on gross area)
E_p = " " " " solid
E_c = " " " " concrete (composite filler and cubes)
a = void area (section through cube) = $1 - F^2$
n = fraction of area subject to uplift
e = strain, inch per inch

$$e_1 = -\frac{Pf}{E_x} - \frac{PF\,(1-2u)}{E_p\,F^2}$$ External load, no pore pressure

$$e_2 = -\frac{P(1-2u)}{E_p}$$ Hydrostatic load with pore pressure

$$e_3 = \frac{nfP}{E_x} + \frac{Pf(2u)}{E_p} + \frac{aPF(1-2u)}{E_p\,F^2}$$ Pore pressure, no external load

The E_x term can be eliminated by expressing it in terms of E_p and E_c as follows: If 1 psi. is applied to layers of filler f inches thick and to a layer of solid F inches thick and F^2 in area, the sum of the deformations is that of one inch of the idealized concrete. Thus the concrete strain for 1 psi. is

$$\frac{1}{E_c} = \frac{f}{E_x} + \frac{F}{F^2\,E_p}$$

Solving for E_x and defining E_p/E_c as C, one finds

$$E_x = \frac{fFE_p}{FC-1}$$

Substituting this value for E_x wherever it appears, and further defining (1-2u) = Q, the equation $e_3 - e_2 = -e_1$, develops as follows:

$$\frac{nf\,(FC-1)}{fFE_p} + \frac{f\,(1-Q)}{E_p} + \frac{aQ}{E_pF} + \frac{Q}{E_p} = \frac{f\,(FC-1)}{fFE_p} + \frac{Q}{FE_p}$$

This can be simplified and solved for n to give

$$n = 1 - \frac{fF}{FC-1}$$

The effective area is now expressed in terms of relative moduli of elasticity and of idealized cell dimensions. The idealized dimensions are in turn dependent upon the relative moduli and the pore volume, and they can be found most readily from Figure 3.

Permeability = 200×10^{-12} cu. ft./sec./sq. ft./ft. head/ft. length.
Porosity = 0.05 (including hydration).
Time in years (t) to penetrate L feet (neglecting capillarity and assuming water incompressible) is:

$$t = 0.01 L^2$$

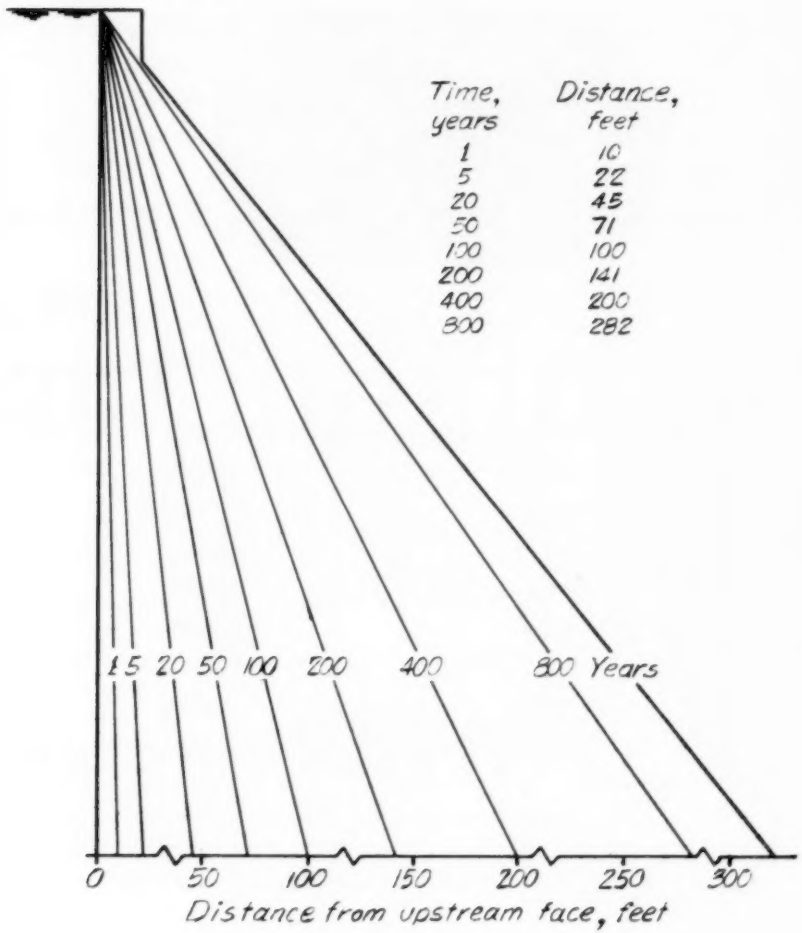
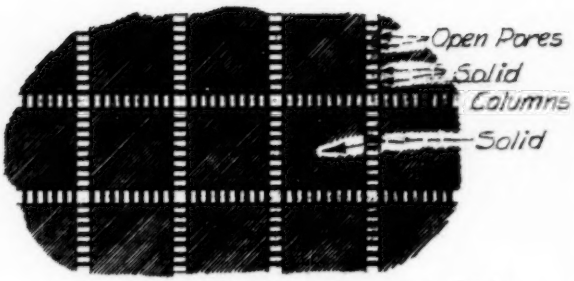
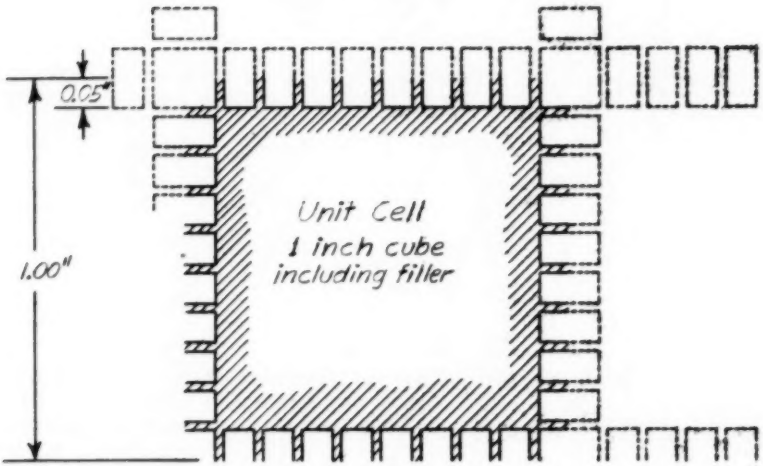


Fig. 1 - Penetration of Water into 400-ft. Dam for Various Durations of Full Reservoir.



Section Showing Several Cells



Enlarged View of Single Cell

Fig. 2- Section of Idealized Concrete.

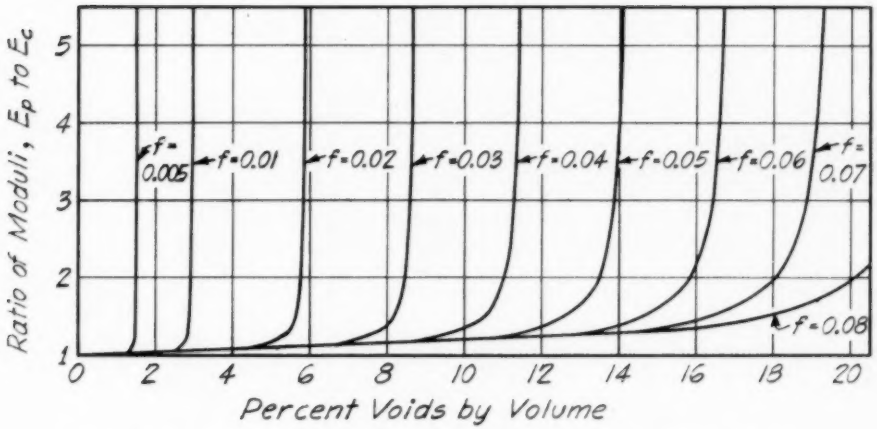


Fig. 3- Chart for finding Thickness of Filler "f" in Idealized Concrete.

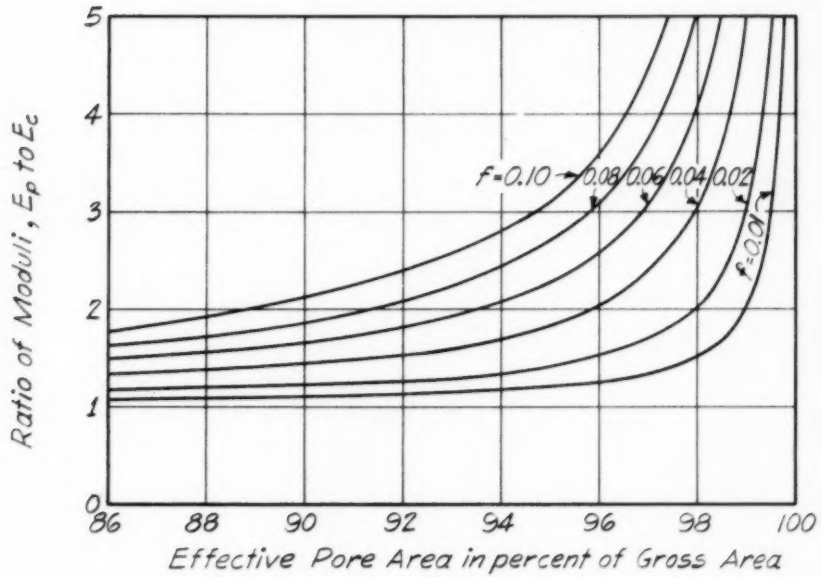


FIG. 4- Effective Pore Area in Idealized Concrete.

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